Synthetic Control Methods to Dynamic Treatment Effects

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Econometrics/Reading Session

Motivation

Motivation

- In the past, our focus has been on quantifying the effects of policy intervention that takes place at an Aggregate level (States, country, Firms).
- Operate in a framework where the treatment occurs at a state-level, i.e., everyone is treated.
- For example Abadie and Gardeazabal, 2003 Study the effect of terrorist conflicts in Basque country on the Basque economy.
- In this setting, we construct the counterfactual using a donor pool of untreated units:
    - Synthetic Control Methods
    - Prediction Model Based Approach.
- This treatment effect obtained is Time-varying.

Is the treatment effect dynamic?

Recap Synthetic Control

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- Suppose that we observe $J+1$ units and only unit 1 is treated at T0.
- The remaining J units are an untreated reservoir of potential controls (a "donor pool").
- We aim to estimate the effect of the intervention on the treated unit,

$$\tau_{1t} = Y_{1t}(D_t = 1) - Y_{1t}(D_t = 0) = Y_{1t} - Y_{1t}^N$$

- The key assumption of SCM is Sequential Treatment Assumption.
- Once treatment starts, the treated unit is continually "exposed" to the treatment. Thus, the treatment effects remain active indefinitely in every subsequent period.
- Even if Y_t is generated by $AR(1)$ process, the treatment effect is still static.

$$Y_t = \rho Y_{t-1} + \tau D_t + \epsilon_t$$

- Hence the treatment effect τ_{1t} is a sequence or collection of static treatment effect.

Dynamic Treatment Effect & SCM

- A treatment effect is dynamic if it explicitly measures how the effect of a one-time intervention unfolds and propagates over multiple periods.
- It does not assume complete or permanent carryover effects.
- Example: Suppose you provide a vaccine once and explicitly track how immunity evolves over time.
- It could be that protection is strong initially but then fades over time.
- This is dynamic because the effect changes over time due to carryover (past effects influence future outcomes).
- However, the SCM framework assumes that the vaccine is injected continuously every day after the initial injection.

Why should we care?

Empirical Example

- Cavallo et al., 2013 analyzes the impact of catastrophic natural disasters (earthquakes, hurricanes) on economic growth.
- The authors use the SCM to create a counterfactual trajectory for countries experiencing these disasters.
- When the SCM method computes the difference at each period, it inherently assumes that the disaster's effect is permanent.
- This inherently implies that the country is permanently in a treated state and the treatment never turns off.
- However, a natural disaster is a one-time shock and the setting at which we interpret this shock as permanent or having a permanent effect does not apply in this setting.

Empirical Example

- Hence, a catastrophic natural disaster does not fit the sequential treatment assumption logically because the disaster itself is not continuously re-occurring.
- Instead, it's a one-time shock whose effects (but not the disaster itself) might propagate dynamically.
- Therefore, our proposed methodology better aligns with the real-world scenario of catastrophic disasters by explicitly modeling the disaster as a one-time shock whose dynamic causal effects unfold naturally over subsequent periods.
- Unlike traditional synthetic controls, our method does not assume continuous exposure, thus offering a more accurate, dynamic depiction of how the initial shock affects growth over time."

Empirical Setting

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- Ouyang and Peng, 2015 studied the treatment effect of the 2008 Chinese Economic Stimulus Program.
- In 2008, the Chinese government put forth an economic stimulus package of four trillion RMB (586 billion USD) as an attempt to minimize the impact of the global financial crisis on China.
- This particular empirical question seeks to evaluate the effect of a one-time treatment over time.
- So clearly, the sequential treatment assumption is not satisfied in this scenario.
- one needs a new framework to evaluate such a one-time treatment effect over time.

Proposal and Idea

Proposal

- Our goal is to extend the synthetic control literature to a panel data setting that estimates the dynamic causal effects of a one-time intervention.
- As pointed out, Traditional Counterfactual imputation methods and the SCM do not estimate dynamic causal effect but instead a collection of a snapshot of the contemporaneous effects.
- Additionally, these methods rely on the sequential treatment assumption, where the treatment is assumed permanent.
- Our proposed methodology, on the other hand, identifies the dynamic causal effect of a one-time intervention at T0 for which after that the treatment turns off.

Proposal + Empirical setting

- This proposed framework fits the empirical settings discussed above,where the catastrophic natural disaster is a one-time shock whose effect on economic growth propagates dynamically over time.
- By allowing the treatment to turn off, we basically assume that any changes we observe in the outcome must be due to the shock or intervention that occurred in T0.
- Therefore, our approach explicitly considers natural disaster a one time shock which is conceptually ’turned off’ after initial implementation, reflecting the true empirical context.

Literature on Dynamic Causal effect

- Identifying Dynamic Treatment effect is a challenging task since the treatment decision depends on past treatment and past outcomes as well.
- As such, a lot of the macroeconomics literature relies on structural modeling and hard restrictions. **christiano1999monetary** ; Bagliano and Favero, 1998
- Our proposed method falls into the class of literature on counterfactual imputation in panel data settings such as Carvalho et al., 2018; Hsiao et al., 2012; Medeiros, 2024.
- In particular, our proposal is inspired by the paper of Carvalho et al., 2018; Hsiao et al., 2012.

Literature

- We also leverage on the work of Bojinov and Shephard, 2019, which builds a potential outcome framework for dynamic settings and defines an estimand of what a dynamic causal effect is.
- our dynamic causal effect methodology also relates to the literature of impulse response functions and local projections literature Angrist et al., 2018; Ballinari and Wehrli, 2024; Gonçalves and Ng, 2024

Framework

Framework

- Suppose we have N units (Countries, Firms, states, etc), indexed as $i = 1, \ldots, N$.
- Let Y_{it} represent the outcome of interest for each unit and for each period t.
- Assume WLOG, Unit 1 receives an intervention $W_t \in \mathcal{W} = \{0, 1\}$ occurs at T_0.
- A static treatment effect is defined as a deterministic sequence such that:

$$Y_{1t}(1) = \begin{cases} Y_{1t}(0); & t = 1, \cdots, T_0 - 1 \\ \tau_t^s + Y_{1t}(0) & t = T_0, \cdots, T \end{cases}$$

- The counterfactual outcome $Y_{1t}(0)$ is imputed using unexposed units in the pre-intervention periods. See Abadie and Gardeazabal, 2003; Abadie et al., 2010; Brodersen et al., 2015; Carvalho et al., 2018; Medeiros, 2024 for details.
- To compute a dynamic causal effect, we need a dynamic causal estimand.

Dynamic Causal Effect

Potential Outcome in Dynamic System

- Let $\{W_t\}_{t=1}^{T}$ represent the treatment path. Given a realization of this treatment path $\{\omega_{1:T}\}$,define a potential outcome as

$$Y_{it}(\omega_{i,1:T}) \quad \forall \quad i = 1, \cdots n \quad t = 1 \cdots T$$

- Assumption 1: **Non-Anticipation**
  For any deterministic $\{\omega_t\}_{t\geq 1}, \{\omega'_t\}_{t\geq 1}$ with $\omega_t, \omega'_t \in \mathcal{W}$;

$$Y_{it}\left(\omega_{1:t}, \{\omega_s\}_{s\geq t+1}\right) = Y_{it}\left(\omega_{1:t}, \{\omega'_s\}_{s\geq t+1}\right) = Y_{it}\left(\omega_{1:t}\right) \quad \forall t \geq 1 \quad \text{a.s}$$

  Under this assumption, the potential outcome path is :

$$\{Y_{1t}(\omega_{1:t})\} = \{Y_{11}(\omega_1), Y_{12}(\omega_{1:2}), \cdots, Y_{1T}(\omega_{1:T})\}$$

- Assumption 2: **Output {Consistency}**
  The output process $\{W_t, Y_{it}\}_{t\geq T_0} = \{W_t, Y_t\left(W_{1:t}\right)\}_{t\geq T_0}$. The $\{Y_t\}_{t\geq 1}$ is called the outcome process.
- Assumption 3: **Sequential Probabilistic assignment:** $0 < P\left(W_t = \omega \mid \mathcal{F}_{t-1}\right) < 1$

Dynamic Causal Estimand

- The dynamic treatment effect at time t is

$$\tau_{it}(\{\omega_{1:t}\}, \{\omega'_{1:t}\}) = Y_{it}(\omega_{1:t}) - Y_{it}(\omega'_{1:t})$$

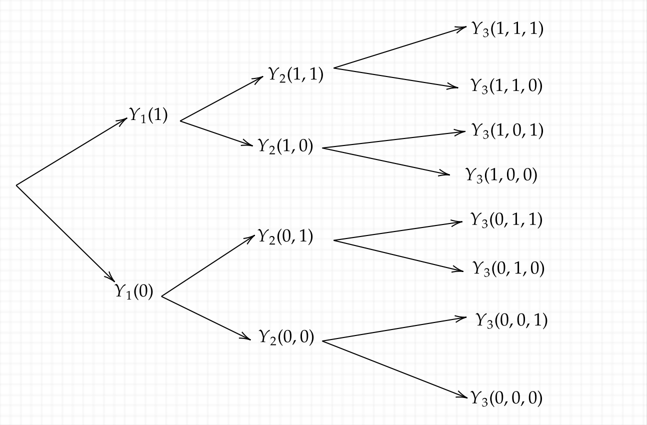
- This defines an enormous class of dynamic causal effects as there are exponentially many possible paths.
- Bojinov and Shephard, 2019 propose isolating the treatment effect at a fixed point in time defined as lag-p causal estimand

$$\tau_{t,p} = Y_t\left(w_{1:t-p-1}, w_{t-p}, w_{t-p+1:t}\right) - Y_t\left(w_{1:t-p-1}, w'_{t-p}, w_{t-p+1:t}\right)$$

- In our framework the dynamic treatment effect:

$$\tau_{1t,T_0} = Y_{1t}\left(\{0\}_{1:T_0-1}, 1, w_{T_0+1:t}\right) - Y_{1t}\left(\{0\}_{1:T_0-1}, 0, w_{T0+1:t}\right) \quad \forall t \geq T_0$$

Dynamic Treatment graph



Numerical Example

- Suppose $Y_{it}\left(\omega_{i,1;t}\right) = \mu_i\left(\omega_{i,1,t:}\right) + \phi\left(\omega_{i,1:t}\right) Y_{it-1}\left(\omega_{i,1,t-1}\right) + \sigma_i\left(\omega_{i,1;t}\right)\varepsilon_{it} \quad \forall i, t$ where $\mu_i\left(w_{i,1:t}\right) = \alpha_i + \Lambda_i F_t + \tau w_{i,t} \quad \phi_i\left(w_{i,1:t}\right) = \phi_i \quad \sigma_i\left(w_{i,1:t}\right) = \sigma$

$$\implies Y_{it}\left(\omega_{i,1;t}\right) = \alpha_i + \lambda_i F_t + \tau w_{i,t} + \phi_i Y_{i,t-1}(w_{1:t-1}) + \sigma\varepsilon_{it}$$

- Unit 1 dynamic treatment effect of a one-time intervention at T0 is given as:

$$\tau_{1,t}(\{1\},\{0\}) = Y_{1t}\left(w_{1:T_0-1}, 1, w_{T_0+1:t}\right) - Y_{1t}\left(w_{1:T_0-1}, 0, w_{T_0+1:t}\right) \quad \forall t = T_0, T_0+1, \ldots, T.$$

$$\begin{aligned}\tau_{1,T0}(\{1\},\{0\}) &= Y_{1,T_0}(\{0\},1) - Y_{1,T_0}(\{0\},0)\\ &= (\alpha_1 + \lambda_1 F_{T_0} + \tau(1) + \phi_1 Y_{1,T_0-1}(\{0\}) + \sigma\varepsilon_{1t})\\ &- ((\alpha_1 + \lambda_1 F_{T_0} + \tau(0) + \phi_1 Y_{1,T_0-1}(\{0\}) + \sigma\varepsilon_{1t})) = \tau\end{aligned}$$

- At $t = T_0 + 1$, the dynamic treatment effect of this one-time intervention will be:

$$\begin{aligned}\tau_{1,T_0+1}(\{1\},\{0\}) &= Y_{1,T_0+1}(w_{1:T_0-1}, 1, w_{T_0+1}) - Y_{1,T_0+1}(w_{1:T_0-1}, 0, w_{T_0+1})\\ &= \tau(w_{T_0+1} = 1) + \phi_1 Y_{1:T0}(\{0\},1) - \left(\tau(w_{T_0+1} = 0) + \phi_1 Y_{1:T0}(\{0\},0)\right)\\ &= \tau + \phi_1(Y_{1:T0}(\{0\},1) - Y_{1:T0}(\{0\},0)) = \tau + \phi_1 \tau_{1,T_0}\end{aligned}$$

Numerical Example

- Hence the dynamic causal effect of the one time intervention is:

$$\tau_{1,t} = \phi^{t-T_0} \cdot \tau_{1,T_0} \quad \forall t \geq T_0$$

- Hence, if we have access to the potential outcomes, one can derive the dynamic causal effect of a one-time intervention at T0 as above.

Identification

Bojinov's Approach

- Although focusing on a one-time treatment effect reduces complexity.
- However at each time t we have 2^{t+1-T_0} potential outcomes for which only one path is observed.
- Under the Randomization and sequential Probability Assignment assumption, Bojinov and Shephard, 2019 proposes the Horvitz Thompson Estimator that identifies the $\tau_{i,t}$ i.e.

$$\hat{\tau}_{1t} = \frac{1}{2^{t-T_0}} \sum_{\omega \in \{0,1\}^{t-T_0}} \left\{ \frac{1_{\{w^{\text{obs}}_{T_0:t} = (1,w)\}}}{p_t(1,w)} Y_t(1,w) - \frac{1_{\{w^{\text{obs}}_{T_0:t} = (0,w)\}}}{p_t(0,w)} Y_t(0,w) \right\}$$

- Methodologies such as Nonparametric Local projections by Gonçalves et al., 2024, semiparametric propensity score approach by Angrist et al., 2018, and Double debiased machine learning approach Ballinari and Wehrli, 2024 .

Counterfactual Imputation Approach

- The goal is to use the rich preintervention data to construct the counterfactuals at each stage.
- In this framework, we bypass the sequential Probability assignment.
- However, we require that once the treatment occurs at time T_0, it turns off after that.
- This identification strategy reduces the dimensionality of the potential outcomes to 2 at each t.
- Our onetime dynamic treatment effect becomes

$$\begin{aligned}\tau_{1t} &= Y_{1,t}(\{0\},1,\{0\}_{t>T_0}) - Y_{1,t}(\{0\},0,\{0\}_{t>T_0}) \quad \forall t \geq T_0 \\ &= Y_{1,t}^{\text{obs}} - Y_{1,t}(\{0\},0,\{0\}_{t>T_0})\end{aligned}$$

- We propose a two-stage estimator for the dynamic treatment effect.
- **Stage 1: Counterfactual Construction via Predictive Modeling.**

$$Y_{it}^0 = \mathcal{M}(\theta, Z_{it}) + u_{it} \tag{1}$$

$$u_{it} = \Lambda_i F_t + \varepsilon_{it} \quad \forall i = 1, \cdots, N \quad \forall t = 1, \cdots T_0 - 1 \tag{2}$$

  where $Y_{it}^0 = Y_{it}(\{w_{it} = 0\})$.
  - Specific Case: Assume $\mathcal{M}(\theta, Z_{it}) = \alpha_i + \phi_i Y_{i,t-1}^0$
- Using the predictive model and the untreated units information, predict

$$\hat{Y}_{1t}^0 = \hat{\alpha}_1 + \hat{\phi}_1 \hat{Y}_{1t-1}^0 + \hat{\Lambda}_1 \hat{F}_t \quad \forall t \geq T_0$$

- **Stage 2: Estimating the Dynamic Causal Effect.**

$$\hat{\tau}_{1t} = Y_{1,t}^{\text{obs}} - \hat{Y}_{1t}^0$$

Simplifying Identification Assumption

- The Factors F_t should be stationary, and F_t must continue to follow the same stochastic process post-treatment.
- The lag coefficient $|\phi_i| < 1$. This ensures model stability.
- $\varepsilon_{it} \sim \mathcal{N}(0, \sigma^2)$.
  - This means no serial or cross-sectional correlation of the errors.
  - This assumption ensures that the effect of the one-time treatment affects future outcomes only through the lag and not through other dependencies.
  - This assumption also implies $\mathbb{E}(\varepsilon_{it}[Z_{it}, F_t]) = 0$
- Assume $\mathbb{E}(FF') = I_r$ and $\mathbb{E}(\Lambda\Lambda') = \textit{diagonal}$
  - This assumption is required to ensure identification of the factors and loading matrix.

Estimation

Estimation

- For simplicity, let $\alpha = 0$.
- The first task is to estimate the model in the pre-intervention period.

$$Y_{it}^0 = \Lambda_i F_t + \phi_i Y_{i,t-1}^0 + \epsilon_{it}$$

Note that we can add covariates such that $X_{it} = [Y_{i,t-1}, Z_{it}]$

- In matrix notation, we have

$$Y = X\beta_i + F\Lambda' + \varepsilon$$

- Note that $F\Lambda' = FAA^{-1}\Lambda' = F^*\Lambda'^*$. Hence, F and Λ are not separately identifiable without restrictions. The normalization $\frac{F'F}{T} = I_r$ and $\Lambda'\Lambda = diagonal$ imposes the r^2 restrictions.
- The goal is to solve the problem:

$$(\hat{\phi}, \hat{F}, \hat{\Lambda}) = \operatorname{argmin} \mathrm{SSR}(\phi, F, \Lambda), \quad SSR = \sum_{t=2}^{T} \sum_{i=1}^{N} (Y_{it} - \phi_i Y_{it-1} - \lambda_i' F_t)^2$$

$$s.t \quad \frac{F'F}{T} = I_r \quad , \quad \Lambda'\Lambda = diagonal$$

Estimation

- The estimation procedure follows the concentration method in Optimization (see Hansen chapter 3).

$$\hat{\phi}_i = argmin \left(\min_{F,\lambda} \mathrm{SSR}(\phi, F, \lambda) \right)$$

- Solve the inner optimization given some guess of ϕ_i such that $W_{it} = Y_{it} - \phi_i Y_{i,t-1}$:

$$\hat{\lambda} = (F'F)^{-1} F'W = \frac{F'W}{T}$$

$$\hat{F} = \sqrt{T}\tilde{V}_R$$

where $\tilde{V}_R$ is the eigenvectos of the first r largest eigenvalues of the covariance WW'

- Obtain the lag coefficient as:

$$\hat{\phi}_i = (Z_i'M_F Z_i)^{-1} Z_i'M_F Y_i = \frac{\sum_{t=2} \left(Y_{it} - \hat{\lambda}_i'\hat{F}_t \right) Y_{it-1}}{\sum Y_{it-1}^2}$$

where $M_F = I_T - F(F'F)^{-1}F'$ and $Z_{it} = Y_{i,t-1}$.

Forecasting

- Practically, we estimate the model iteratively, i.e.
  - Initialise ϕ_i, and compute $W_{it} = Y_{it} - \phi_i Y_{i,t-1}$
  - Using the Principal Component Analysis Approach, estimate F_t and λ_i
  - Update the ϕ_i by running OLS.
- To forecast the counterfactual for unit 1 for $t \geq T_0$ $\hat{Y}_{1t} = \hat{\lambda}_i \hat{F}_t + \hat{\phi}_i \hat{Y}_{1,t-1}$, we need a forecast for $\hat{F}_t$
- Forecast the post intervention factors using the untreated units post-treatment data i.e.

$$\hat{F}_t = (\hat{\lambda}_i' \hat{\lambda}_i)^{-1} \hat{\lambda}_i (Y_{it} - \hat{\phi}_i Y_{i,t-1}) \quad \forall i = 2, ...N \quad , t = T_0, ..., T$$

- $\hat{\tau}_{1,t} = Y_{1,t}^{\text{obs}} - \hat{Y}_{1t}^{0}$

Asymptotic Properties

Consistency

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- Given that $\hat{\tau}_{1,t} - \tau_{1,t} = Y^0_{1,t} - \hat{Y}^0_{1,t} \quad \forall t \geq T_0$
- This means that any uncertainty of the DCE estimator comes solely from the predictive model.

$$\hat{\tau}_{1,t} - \tau_{1,t} = (\lambda_1' F_t - \hat{\lambda}_1' \hat{F}_t) + (\phi_1 Y_{1,t-1} - \hat{\phi}_1 \hat{Y}_{1,t-1}) + \varepsilon_{1t} \quad \forall t \geq T_0$$

- The consistency of the DCE estimator depends on the consistency of estimated parameters in the pre-intervention period but also depends on the forecasted variables F_t and $Y_{1,t-1}$ for $t > T_0$.
- Claim: If $\hat{\phi}_1 \to \phi_1, \quad \hat{\lambda}_1 \to \lambda_1 \quad$ and $\quad \hat{F}_t \to F_t$ for $t < T_0$ then $\hat{\tau}_{1,t}$ is consistent.
- By recursive nature of the model,

$$Y^0_{i,t} = \phi_i Y_{i,t-1} + \lambda_i F_t + \varepsilon_{it} \quad \forall t \geq T_0$$

there exist a function $G : (\phi_1, \lambda_1, \{F_t\}_{t \geq T_0}, Y_{T_0-1}) \to (Y^0_{1,T_0}, Y^0_{1,T_0+1}, \dots)$

Consistency

- The sketch of the proof relies on the Continous Mapping Theorem.
- If $G : (\phi_1, \lambda_1, \{F_t\}_{t \geq T_0}, Y_{T_0 - 1}) \rightharpoonup (Y^0_{1,T_0}, Y^0_{1,T_0+1}, \ldots)$ is continous in the parameters and given that

$$(\hat{\phi}_1, \hat{\lambda}_1, \hat{F}_t) \rightarrow (\phi, \lambda_1, F_t) \implies$$
$$G(\hat{\phi}_1, \hat{\lambda}_1, \{\hat{F}_t\}) \rightarrow G(\phi_1, \lambda_1, \{F_t\}) \quad \forall t \geq T_0$$
$$\text{Hence} \quad \hat{Y}_{1,t} \rightarrow Y_{1,t} \quad \text{for each fixed t}$$

- By Bai, 2009, the parameters are consistent under both large T and large N.
- Additionally, since G is constructed based on the recursiveness of the model, if $\phi_1 < 1$, i.e., under stationarity, G should be continuous.

Asymptotic Normality

- Let $\theta = (\phi_1, \lambda_1, \{F_t\}_{t \geq T_0})$
- Redefine the map G as $G(\theta, Y^0_{T_0-1}) = \{Y_{1,t}\}_{t \geq T_0}$ and $\hat{Y}^0_{1,t} = G(\hat{\theta}, Y^0_{T_0-1})$
- By assumption the pre-treatment estimators are consistent and asymptotically normal:

$$\sqrt{NT}\left(\hat{\theta} - \theta_0\right) \Rightarrow N\left(0, \Sigma_\theta\right)$$

- If G is continuously differentiable with respect to θ, the delta method implies

$$\sqrt{NT}\left(G\left(\hat{\theta}, Y_{1,T_0-1}(0)\right) - G\left(\theta_0, Y_{1,T_0-1}(0)\right)\right) = G'\left(\theta_0\right)\sqrt{NT}\left(\hat{\theta} - \theta_0\right) + o_p(1)$$

- Which implies

$$\sqrt{NT}\left(\widehat{Y}_{1,t}(0) - Y_{1,t}(0)\right) \Rightarrow N\left(0, \sigma^2_{Y,t}\right)$$

with asymptotic variance

$$\sigma^2_{Y,t} = G'\left(\theta_0\right)\Sigma_\theta G'\left(\theta_0\right)^\top$$

Simulations

DGP

- The goal of the simulation is to evaluate some basic properties of the estimator such as Bias, consistency, and Asymptotic distribution.
- We simulate the data for the baseline case, where no one receives the treatment:

$$Y_{it}^{0} = \alpha_i + \Lambda_i F_t + \phi_i Y_{i,t-1}^{0} + \varepsilon_{it} \quad \forall i = 1, \cdots, N \quad \forall t = 1, \cdots T$$

- Factors are generated according to $F_t \sim N(0, \sigma_F)$ and $F_t = \rho_F F_{t-1} + v_t$, where $v_t \sim iid \quad N(0, 1)$
- $\epsilon_{it} \sim N(0, \sigma)$, $\phi_i = \rho * A_i$
- Since only unit 1 is treated, we update the unit 1 outcome using

$$Y_{1t}(\{0\}, 1, \{0\}) = Y_{1t}^{(0)} + \tau \phi_1^{t-T_0} w_{T_0}$$

Simulation Results

Case 1: iid errors, normal Factors, T = 1000, N = 100

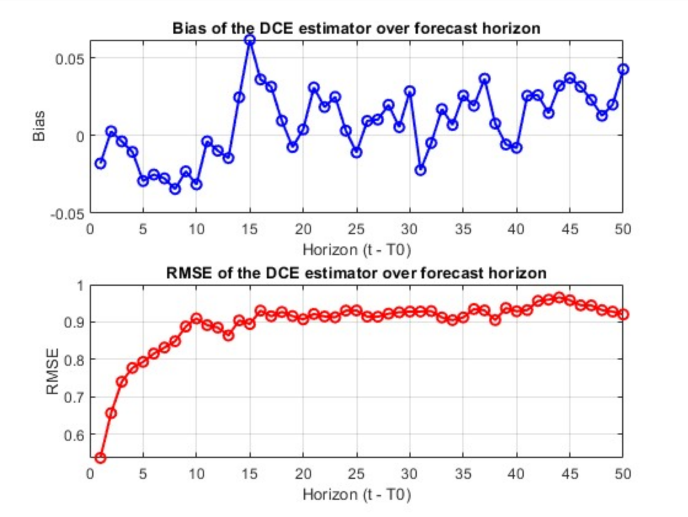


Figure 2: Caption

Case 1: iid errors, normal Factors, T = 1000

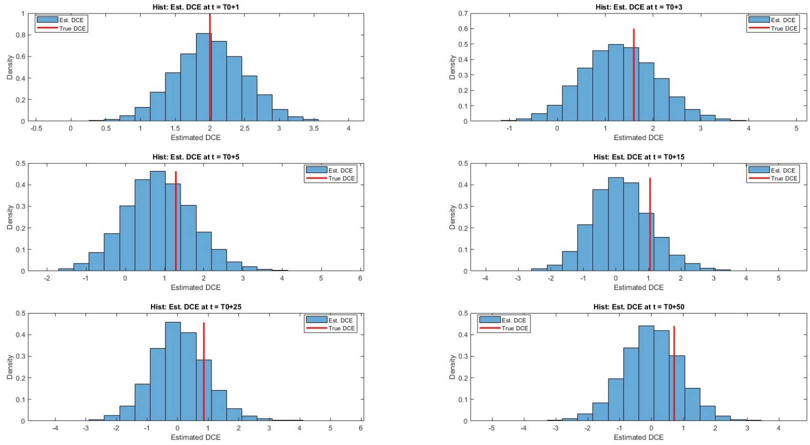


Figure 3: Caption

Case 1: iid errors, normal Factors, T = 1000

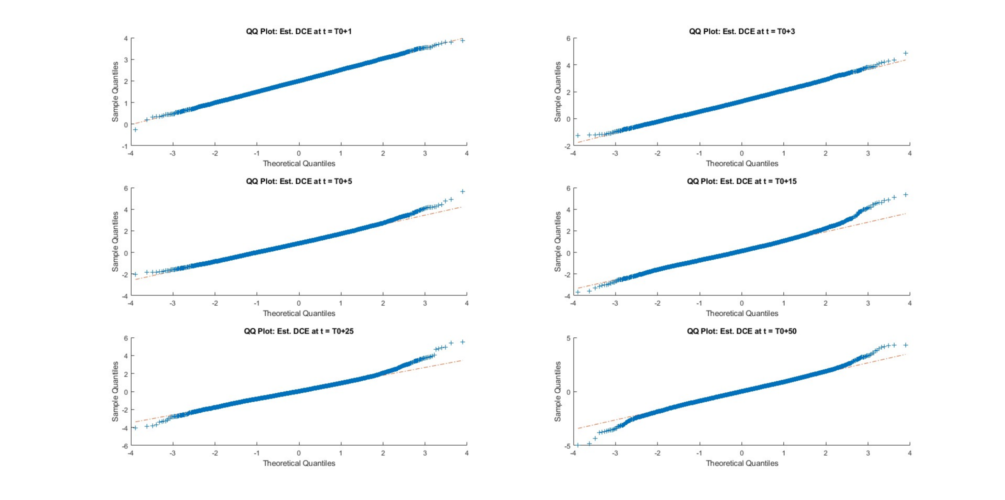


Figure 4: Caption

Dynamic Factor: Ft   AR(1)

Case 2: iid errors, AR Factors, T = 1000, N = 100

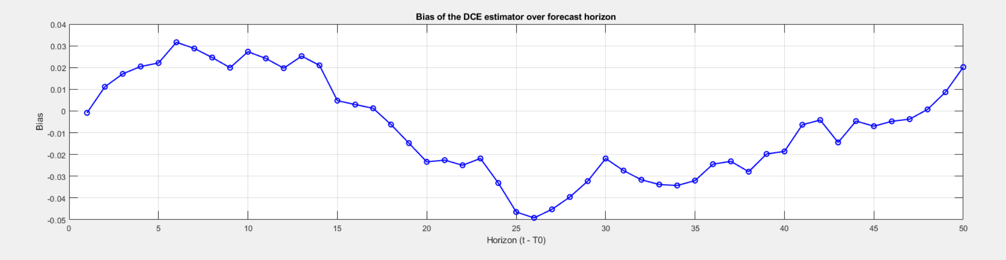


Figure 5: Caption

Case 2: iid errors, AR Factors, T = 1000, N = 100

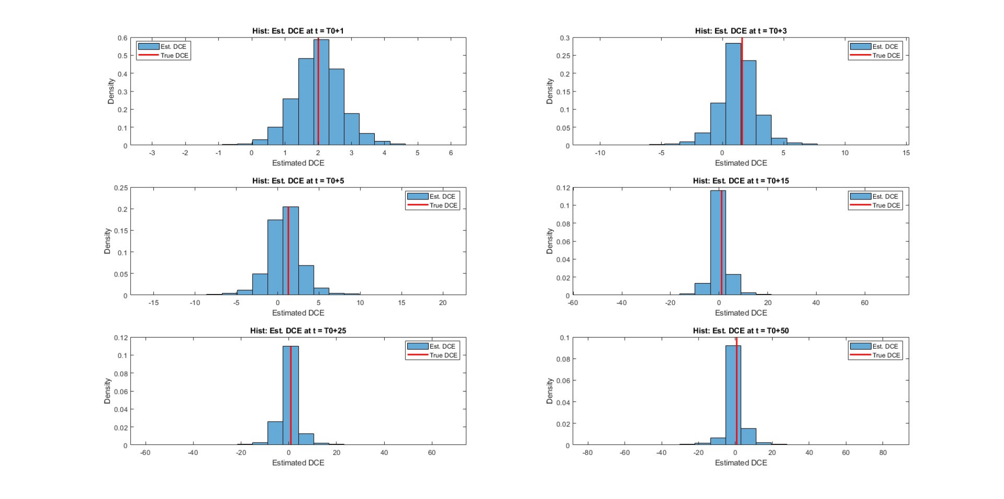


Figure 6: Caption

Case 2: iid errors, AR Factors, T = 1000, N = 100

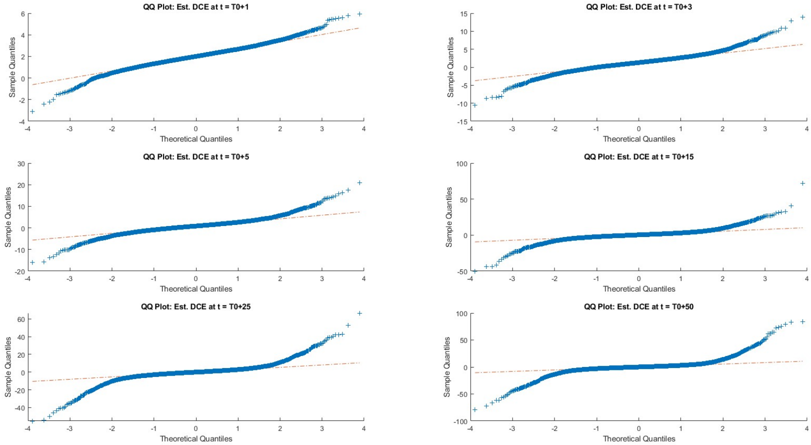
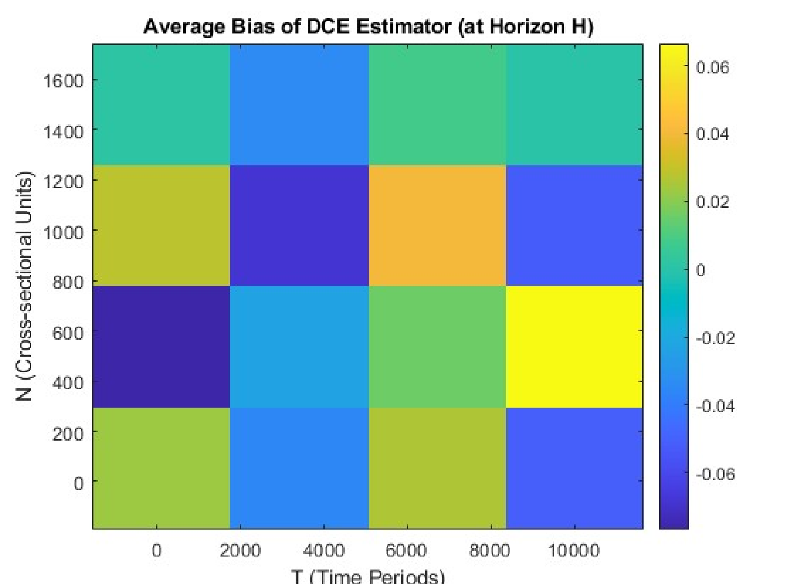


Figure 7: Caption

Case 3: iid errors, normal Factors

N	T	**Bias**	**RMSE**
50	100	0.0231	0.8931
50	500	-0.0351	0.9227
50	1000	0.0258	0.9827
50	10000	-0.0512	0.9749
200	100	-0.0768	0.8866
200	500	-0.0232	0.9337
200	1000	0.0157	0.9528
200	10000	0.0665	0.9730
800	100	0.0280	0.8301
800	500	-0.0680	0.9332
800	1000	0.0406	0.9157
800	10000	-0.0518	0.9280
1500	100	0.0013	0.8072
1500	500	-0.0339	0.8465
1500	1000	0.0081	0.9535

Heat Map of Average Bias for N and T



Correlated Errors

Case 3: Correlated errors

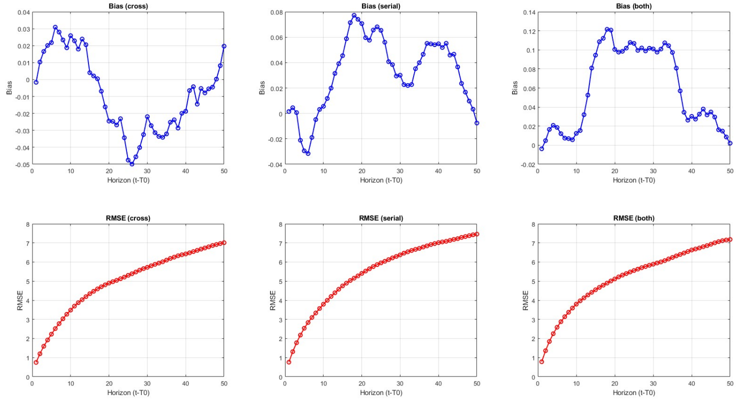


Figure 9: Caption

Case: 3 correlated Errors

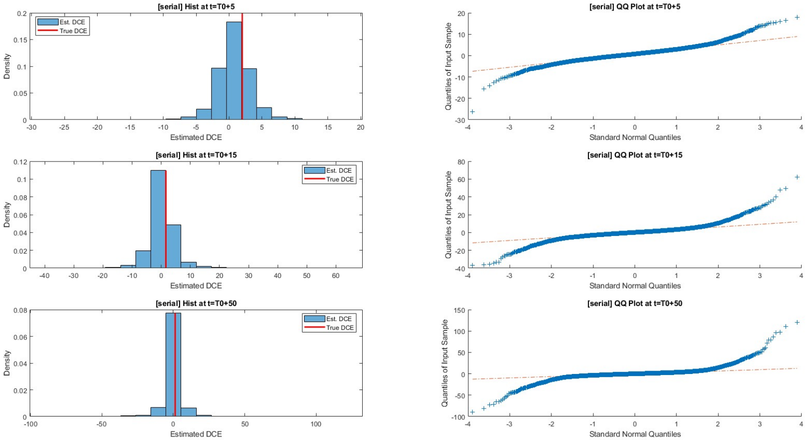


Figure 10: Caption

Case: 3 correlated Errors

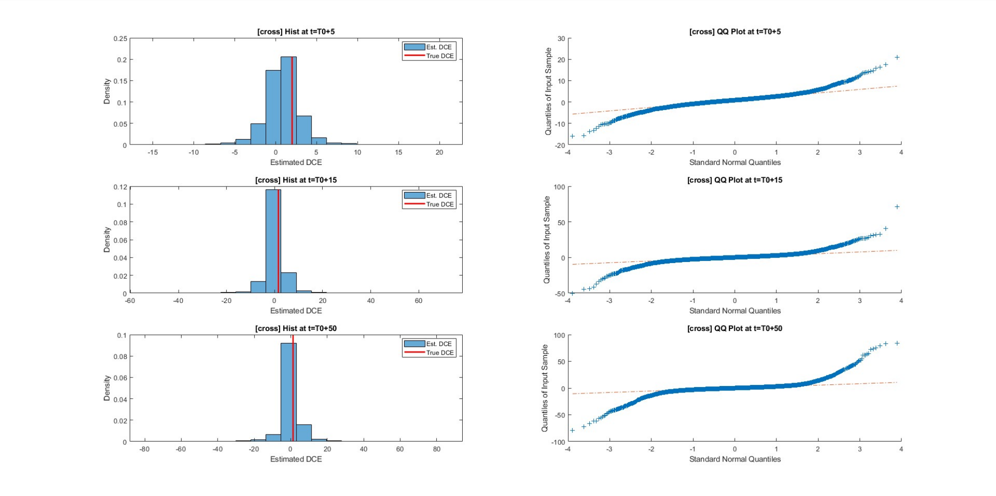


Figure 11: Caption

Case: 3 correlated Errors

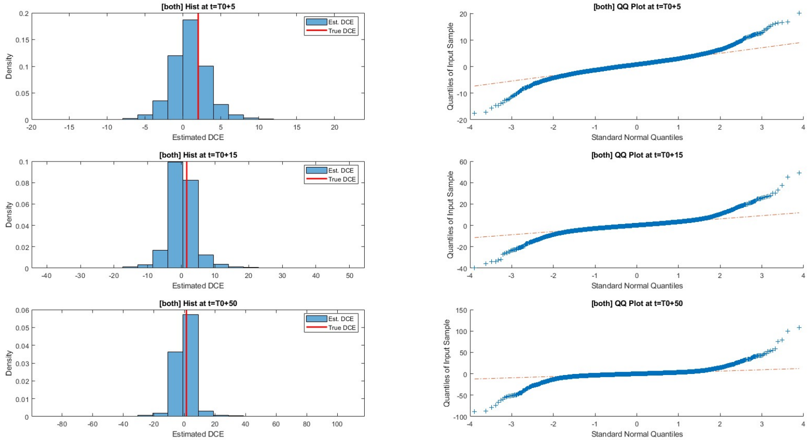


Figure 12: Caption

Conclusion

Thanks!

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